

Trade of Electrician

Standards Based Apprenticeship

Resistance Network Measurement

Phase 2

Module No. 2.1

Unit No. 2.1.5

COURSE NOTES

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Introduction

Welcome to this section of your course which is designed to assist you the learner, understand more practical electrical circuits and complete important circuit calculations.

Objectives

By the end of this unit you will be able to:

- Connect resistors in series
- Calculate total resistance of resistors in series
- Connect resistors in parallel
- Calculate total resistance of resistors in parallel
- Connect resistors in series-parallel
- Calculate total resistance of resistors in series-parallel
- Understand the loading effect of an analogue voltmeter
- Connect cells in series and calculate output voltage
- Connect cells in parallel and calculate output voltage
- Measure output voltage of a battery
- Understand internal resistance of cells
- Understand resistivity
- Calculate conductor resistance
- Understand, temperature coefficient of resistance

Reasons

Understanding the information in this unit will give you the confidence to build and solve problems on practical electrical circuits.

Electrical Circuits

Electrical circuits fall into three categories:

1. The Series Circuit
2. The Parallel Circuit
3. The Series Parallel Circuit

The Series Circuit

In a series circuit there is only one path for current to flow. This path must lead from the supply source, through all the resistance units or circuit components and return to the source. In this unit and other units we will use the “**conventional theory of current flow**”.

Where two or more resistors, are connected **end to end**, they are said to be connected in **series**. Figure 1 shows three resistors connected in series in a circuit. Note that there is only one path through which current can flow.

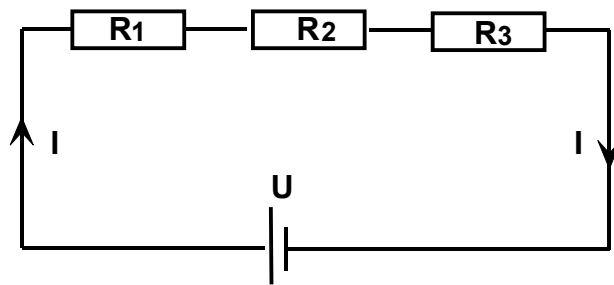


Figure 1.

The formula used to find the **Total Resistance** (R_T) for the circuit is shown below.

Formula

$$R_T = R_1 + R_2$$

With **3** resistors in series: $R_T = R_1 + R_2 + R_3$

If **N** resistors in series: $R_T = R_1 + R_2 + \dots + R_N$

R_T is sometimes referred to as the “**equivalent resistance**” of the circuit.

Example:

Refer to Figure 2. What is the total resistance of this circuit?

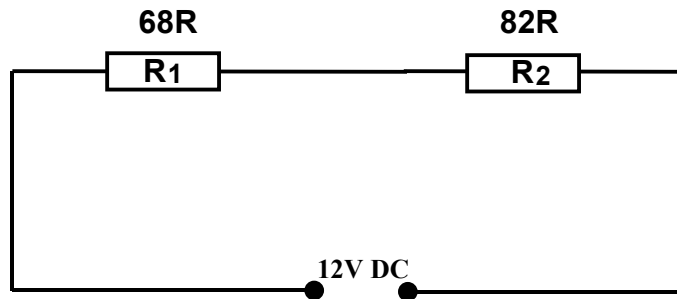


Figure 2.

Solution:

$$R_T = R_1 + R_2$$

$$R_T = 68 + 82$$

$$R_T = \underline{\underline{150 \Omega}}$$

This value of 150R is the total resistance or equivalent resistance of the series circuit. The total resistance (R_T) of a series circuit is always **greater than** the largest resistor (82 R) in the circuit.

Using Ohm's Law the current can be calculated.

$$I = \frac{U}{R_T} \quad (U = 12 \text{ V})$$

$$\quad \quad \quad (R_T = 150 \Omega)$$

$$I = \frac{12}{150}$$

$$I = \underline{\underline{0.08 \text{ Amp or } 80 \text{ mA}}}$$

Note: In a series circuit there is only one path for current to flow, so 80 mA will flow through the 68 Ω and the 82 Ω resistor.

Activity:

Apprentices be given two resistors, 100R and 150R

1. Use the resistor colour code to check each resistor and note their values.
2. Use a multimeter to measure the value of each resistor and check against coded value and tolerance.
3. Refer to Figure 3 and calculate the total circuit resistance.
4. Connect resistors in series and measure the total resistance with an ohmmeter and compare with the calculated value.
5. Calculate the circuit current assuming a supply voltage of 6 Volts DC.
6. Connect up the circuit shown in Figure 4, and measure the circuit current.
7. Check measured values, against calculated values. List reasons for difference if any.

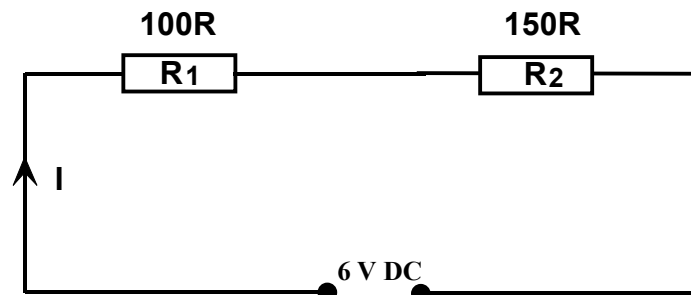


Figure 3.

Refer to Figure 4

Apprentice to measure the current flowing in the circuit using a multimeter and / or an ammeter.

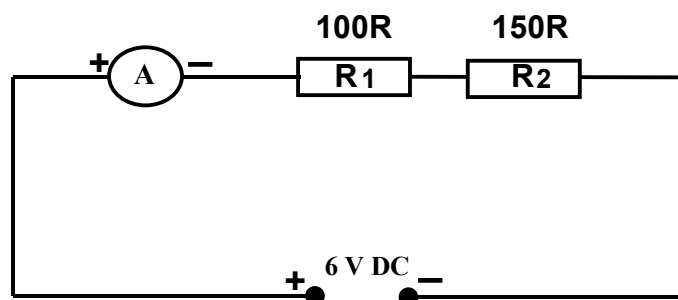


Figure 4.

Series Circuit Calculations

Example

Three resistors of 270R, 330R, and 4k7 values are connected in **series**. Calculate the equivalent resistance.

$$\begin{aligned}
 R_T &= R_1 + R_2 + R_3 \\
 R_T &= 270R + 330R + 4k7 \\
 R_T &= 270 + 330 + 4,700 \\
 R_T &= \underline{\underline{5,300R \text{ or } 5k3}}
 \end{aligned}$$

Refer to Figure 5.

Calculate the total resistance and the total current of the circuit.

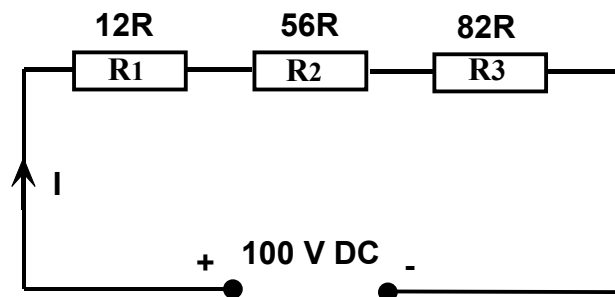


Figure 5.

$$\begin{aligned}
 R_T &= R_1 + R_2 + R_3 \\
 R_T &= 12R + 56R + 82R \\
 R_T &= 12 + 56 + 82 \\
 R_T &= \underline{\underline{150R}} \\
 \\
 I &= \frac{U}{R_T} \\
 I &= \frac{100}{150} \\
 I &= \underline{\underline{0.666 \text{ Amperes or } 666 \text{ mA}}}
 \end{aligned}$$

Series Circuit Volt Drop

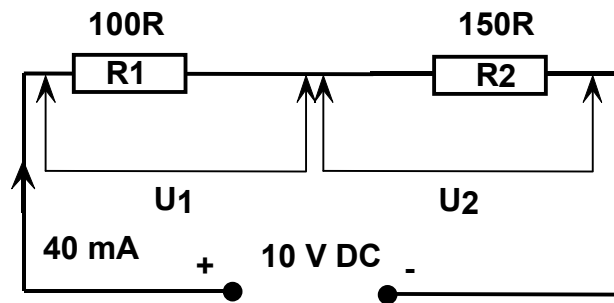


Figure 6.

Refer to Figure 6. A 40 mA current flows through BOTH the 100R resistor and the 150R resistor. If we wish to calculate the volt drop across each resistor we can apply Ohm's Law.

Notice that U_1 is across R_1 and that U_2 is across R_2

$$U_1 = I \times R_1$$

$$U_1 = 0.04 \times 100 \quad (40 \text{ mA} = 0.04 \text{ A})$$

$$U_1 = \underline{\mathbf{4 \text{ Volts.}}} \quad (\text{There is a volt drop of 4 Volts across the resistor } R_1).$$

$$U_2 = I \times R_2$$

$$U_2 = 0.04 \times 150$$

$$U_2 = \underline{\mathbf{6 \text{ Volts.}}} \quad (\text{There is a volt drop of 6 Volts across the resistor } R_2).$$

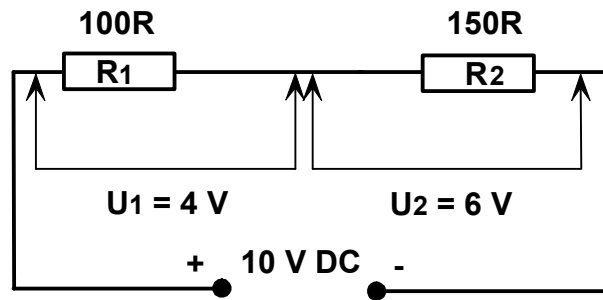


Figure 7.

In a **series circuit** the sum of all the volt drops is equal to the applied voltage (U_A).

$$U_1 + U_2 = U_A$$

or
$$U_A = U_1 + U_2$$

or
$$10 \text{ Volts} = 4 \text{ Volts} + 6 \text{ Volts.} \quad \text{See Figure 7.}$$

Note:

The larger volt drop is across the larger value resistor.

The smaller volt drop is across the smaller value resistor.

The volt drops are proportional to the values of the resistors.

Activity

Refer to Figure 8. Apprentices to measure the volt drop across each resistor.

Reminder:

1. Are the meter test lead(s) in appropriate jack sockets with correct polarity?
2. Is the meter set properly to measure voltage?
3. Is the meter set to the correct voltage range?
4. Measure the applied voltage.
5. Measure the volt drop across each resistor.
6. Compare measured voltages with calculated values.

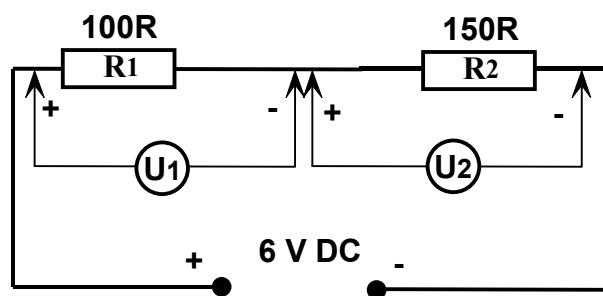


Figure 8.

Activity

Refer to Figure 9. Apprentices to:

1. Connect two lamps in series and measure the volt drop across each lamp.
2. Remove one lamp and observe what happens to the other.

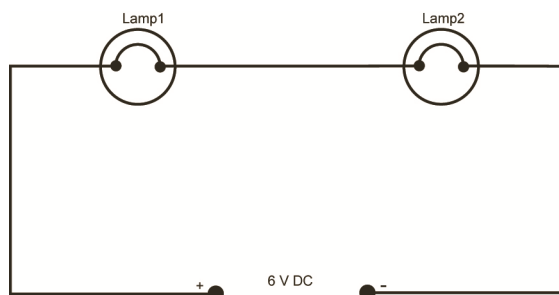


Figure 9.

Question 1

Refer to Figure 10. Find volt drops U_1 and U_2

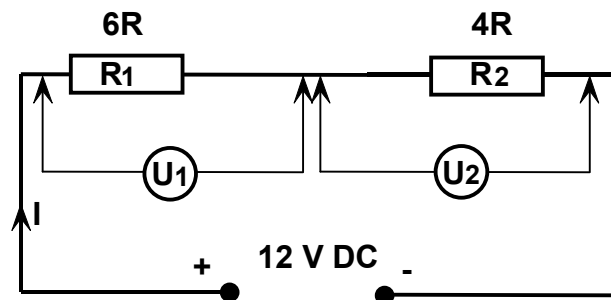


Figure 10.

Solution: $R_T = R_1 + R_2$

$$R_T = 6 + 4$$

$$R_T = \underline{10 \Omega}$$

$$I = \frac{U}{R_T} = \frac{12}{10} = \underline{1.2 \text{ Amps}}$$

Volt drop $U_1 = I \times R_1 = 1.2 \times 6 = \underline{7.2 \text{ Volts}}$

Volt drop $U_2 = I \times R_2 = 1.2 \times 4 = \underline{4.8 \text{ Volts}}$

Note:

The sum of all the volt drops in a series circuit is equal to the Applied Voltage.

Question 2

Refer to Figure 11. Calculate the value of R_3

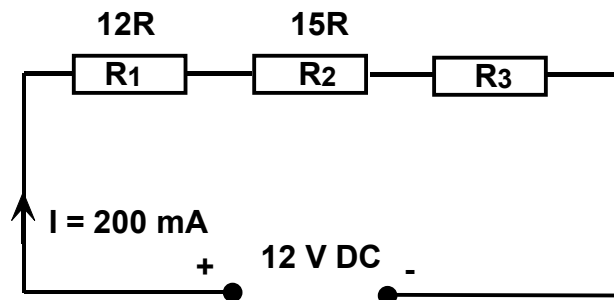


Figure 11.

Solution: There will be three, volt drops: U_1 across R_1
 U_2 across R_2
 U_3 across R_3

$$I = 200 \text{ mA} = 0.2 \text{ Amps}$$

From Ohm's Law:

$$\text{Volt drop across } R_1 (U_1) = I \times R_1 = 0.2 \times 12 = \underline{\underline{2.4 \text{ Volts}}}$$

$$\text{Volt drop across } R_2 (U_2) = I \times R_2 = 0.2 \times 15 = \underline{\underline{3.0 \text{ Volts}}}$$

$$\begin{aligned} \text{In a series circuit: } U_A &= U_1 + U_2 + U_3 \\ 12 &= 2.4 + 3.0 + U_3 \\ 12 &= 5.4 + U_3 \\ U_3 &= 12 - 5.4 \\ U_3 &= \underline{\underline{6.6 \text{ Volts}}} \end{aligned}$$

$$R_3 = \frac{U_3}{I}$$

$$R_3 = \frac{6.6}{0.2}$$

$$R_3 = \underline{\underline{33 \Omega}}$$

Summary of the Series Circuit

1. In a series circuit the same current flows through all resistors.

$$I_T = I_1 = I_2 = I_3 = \dots I_N$$

2. If there is a break in the circuit no current flows in **any** part of the circuit.
3. The total resistance is the sum of all resistors in the series circuit.

$$R_T = R_1 + R_2 + R_3 + \dots R_N$$

4. The sum of all the volt drops in the circuit is equal to the **applied voltage**.

$$U_A = U_1 + U_2 + U_3 + \dots U_N$$

5. The **volt drops** are proportional to the values of the resistors in the circuit.
6. An example of a series circuit is a Christmas tree lighting set.

Sample Questions:

1. Four resistors of values 5 Ω , 15 Ω , 20 Ω and 40 Ω , respectively, are connected in series to a 230 V supply. Calculate the resulting current and the volt drop across each resistor.
2. A 6 Ω resistor and a resistor of unknown value are connected in series to a 12 V supply, and the volt drop across the 6 Ω resistor is 9 V. What is the value of the unknown resistor?
3. Calculate the resistance of the element of a soldering iron that takes 0.5 A from 230 V mains when connected by cables having a total resistance of 0.2 Ω .

The Parallel Circuit

A circuit with two or more paths through which current can flow, is called a **parallel circuit**.

Refer to Figure 12. The total current (I_T) divides itself among the resistors, so that:

$$I_T = I_1 + I_2 + I_3$$

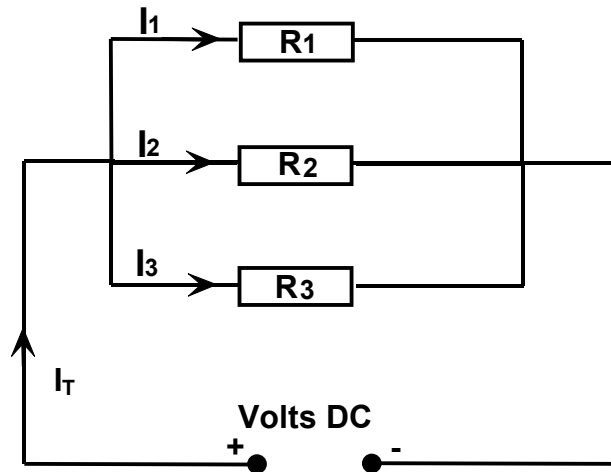


Figure 12.

From Ohm's Law

$$I_1 = \frac{U}{R_1} \quad I_2 = \frac{U}{R_2} \quad I_3 = \frac{U}{R_3}$$

The total resistance R_T of a parallel circuit is found by the following formula:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

The value $\frac{1}{R}$ is called the **reciprocal** of R.

The reciprocal, of a parallel circuit total resistance, is equal to the sum of the reciprocals, of the individual resistors in the circuit.

Parallel Circuit Calculations

Example:

Refer to Figure 13. What is the total resistance of this circuit?

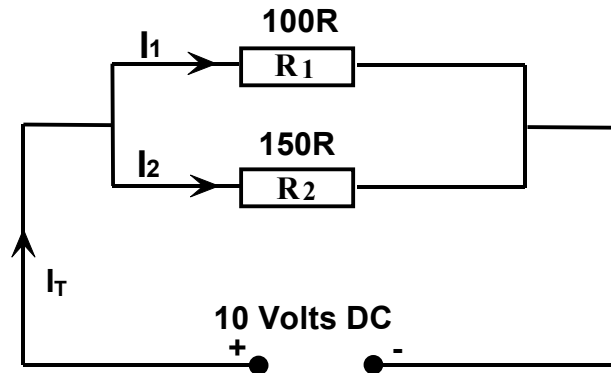


Figure 13.

To find R_T

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_T} = \frac{1}{100} + \frac{1}{150}$$

$$\frac{1}{R_T} = \frac{3}{300} + \frac{2}{300} = \frac{5}{300}$$

$$R_T = \frac{300}{5}$$

$$R_T = \underline{\underline{60 \text{ Ohms}}}$$

This value of 60R is the total resistance or equivalent resistance of the parallel circuit. The total resistance (R_T) of a parallel circuit is always **less than** the smallest resistor (100R) in the circuit.

Refer again to Figure 13. The total current I_T can be calculated in two ways:

First Method:

Calculate I_1 and I_2

$$I_1 = \frac{U}{R_1}$$

$$I_1 = \frac{10}{100} \quad (\text{Voltage same in parallel circuit})$$

$$I_1 = \underline{\underline{0.1 \text{ Amp}}}$$

$$I_2 = \frac{U}{R_2}$$

$$I_2 = \frac{10}{150}$$

$$I_2 = \underline{\underline{0.0667 \text{ Amp}}} \quad (\text{to 4 decimal places})$$

$$I_T = I_1 + I_2$$

$$I_T = 0.1 + 0.0667$$

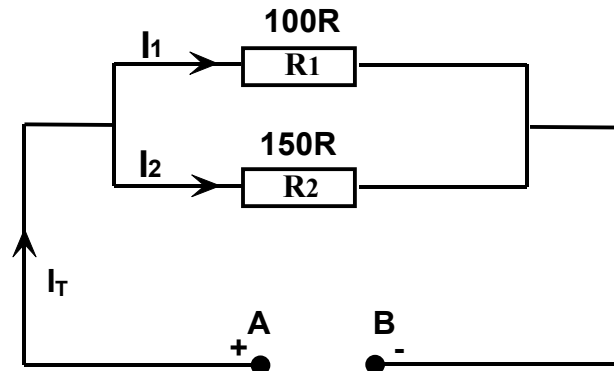
$$I_T = \underline{\underline{0.1667 \text{ Amps}}} \text{ or } 166.7 \text{ mA}$$

Second Method:

$$I_T = \frac{U}{R_T}$$

$$I_T = \frac{10}{60}$$

$$I_T = \underline{\underline{0.1667 \text{ Amps}}} \text{ or } 166.7 \text{ mA}$$

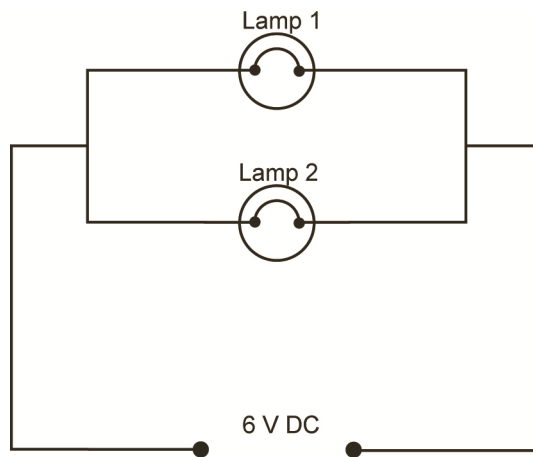
Activity 1**Figure 14.**

Refer to Figure 14.

Apprentice to complete the following:

1. Measure the total resistance at points A and B (compare with calculated value)
2. Connect 6 Volts DC, the Positive to A and Negative to B and measure the supply voltage with a meter.
3. Measure the voltage across the 100R.
4. Measure the voltage across the 150R.
5. Draw a circuit showing an ammeter connected to measure the current I_1 .
6. Measure the current I_1 (break the circuit and connect an ammeter or multi-meter).
7. Measure the current I_2 (as above)
8. Measure the total current I_T (as above)

Does $I_T = I_1 + I_2$?

Activity 2**Figure 15.**

Refer to Figure 15.

Apprentices to:

1. Connect two lamps in parallel and measure the voltage across each lamp.
2. Remove one lamp and observe what happens to the other lamp.

Example:

For Figure 16 find the total resistance (R_T) and the total current (I_T)

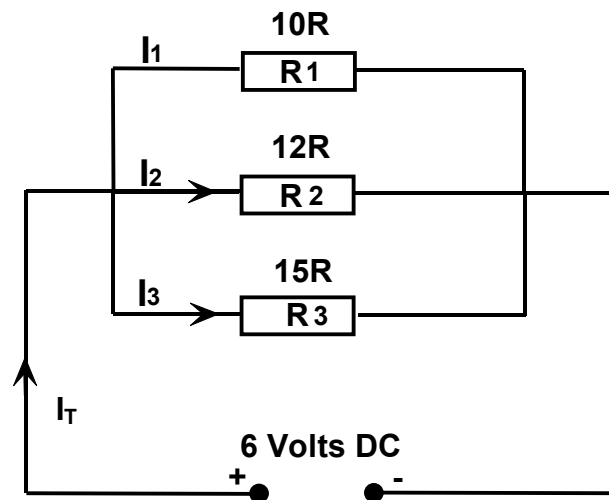


Figure 16.

Solution:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_T} = \frac{1}{10} + \frac{1}{12} + \frac{1}{15}$$

$$\frac{1}{R_T} = \frac{6}{60} + \frac{5}{60} + \frac{4}{60}$$

$$\frac{1}{R_T} = \frac{15}{60}$$

$$R_T = \frac{60}{15} = \underline{\underline{4\ \Omega}}$$

$$I_T = \frac{U}{R_T} = \frac{6}{4} = \underline{\underline{1.5\ \text{Amps}}}$$

Calculate:

I_1 , I_2 and I_3

Example:

Refer to Figure 17. Find I_1 , I_2 , I_3 and I_T

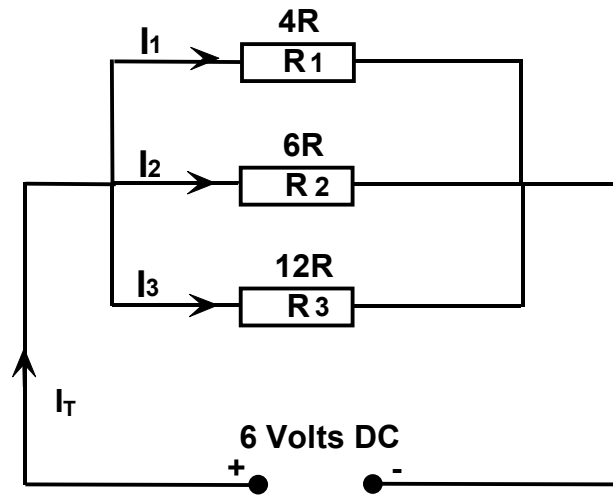


Figure 17.

Solution:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_T} = \frac{1}{4} + \frac{1}{6} + \frac{1}{12}$$

$$\frac{1}{R_T} = \frac{3 + 2 + 1}{12}$$

$$\frac{1}{R_T} = \frac{6}{12}$$

$$R_T = \frac{12}{6}$$

$$R_T = \underline{\underline{2\Omega}}$$

$$I_T = \frac{U}{R_T}$$

$$I_T = \frac{6}{2}$$

$$I_T = \underline{\underline{3 \text{ Amps}}}$$

$$I_1 = \frac{U}{R_1} = \frac{6}{4} = \underline{\underline{1.5 \text{ Amps}}}$$

$$I_2 = \frac{U}{R_2} = \frac{6}{6} = \underline{\underline{1.0 \text{ Amp}}}$$

$$I_3 = \frac{U}{R_3} = \frac{6}{12} = \underline{\underline{0.5 \text{ Amp}}}$$

Check:

$$I_T = I_1 + I_2 + I_3$$

$$I_T = 1.5 + 1.0 + 0.5$$

$$I_T = \underline{\underline{3.0 \text{ Amps}}}$$

Example:

Refer to Figure 18. Find the supply voltage.

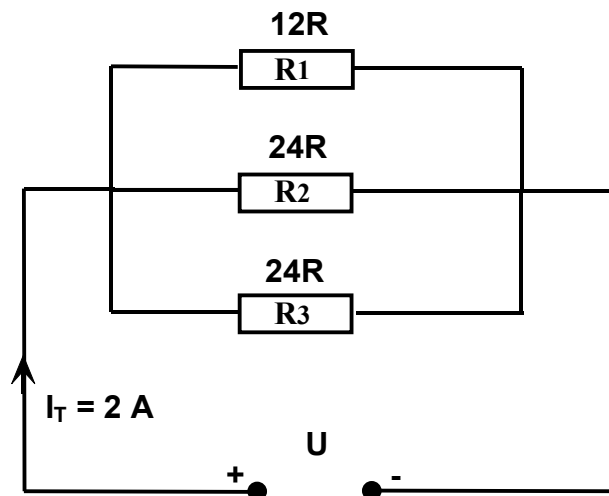


Figure 18.

Solution

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_T} = \frac{1}{12} + \frac{1}{24} + \frac{1}{24}$$

$$\frac{1}{R_T} = \frac{2 + 1 + 1}{24}$$

$$\frac{1}{R_T} = \frac{4}{24}$$

$$R_T = \frac{24}{4}$$

$$R_T = \underline{\underline{6\ \Omega}}$$

$$U = I_T \times R_T$$

$$U = 2 \times 6$$

$$U = \underline{\underline{12\ \text{Volts}}}$$

Calculate:

I_1 , I_2 and I_3

Equal Resistors in Parallel

Refer to Figure 19. Where a number of **equal value** resistors are connected in parallel, their total resistance can be calculated as follows:

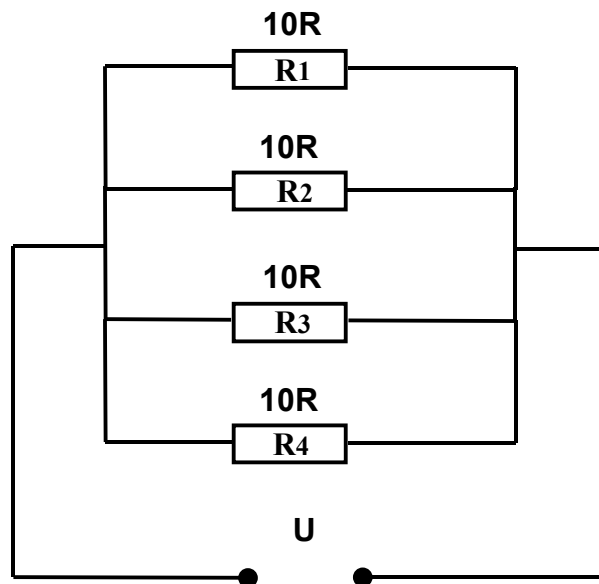


Figure 19.

$$R_T = \frac{\text{The value of one resistor}}{\text{The number of resistors}} = \frac{10}{4} = \underline{\underline{2R5}}$$

Check:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

$$\frac{1}{R_T} = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10}$$

$$\frac{1}{R_T} = \frac{4}{10}$$

$$R_T = \frac{10}{4} = \underline{\underline{2R5}}$$

Two Unequal Resistors in Parallel

Refer to Figure 20. When **two** unequal resistors are connected in parallel, their total resistance can be calculated as follows:

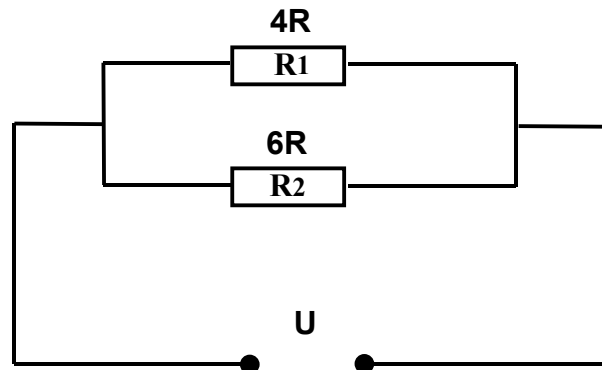


Figure 20.

$$R_T = \frac{\text{Product}}{\text{Sum}} = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{6 \times 4}{6 + 4} = \frac{24}{10} = \underline{\underline{2R4}}$$

Check:

$$\begin{aligned} \frac{1}{R_T} &= \frac{1}{R_1} + \frac{1}{R_2} \\ \frac{1}{R_T} &= \frac{1}{4} + \frac{1}{6} \\ \frac{1}{R_T} &= \frac{3}{12} + \frac{2}{12} \\ \frac{1}{R_T} &= \frac{5}{12} \\ R_T &= \frac{12}{5} = \underline{\underline{2R4}} \end{aligned}$$

Open Circuit Fault

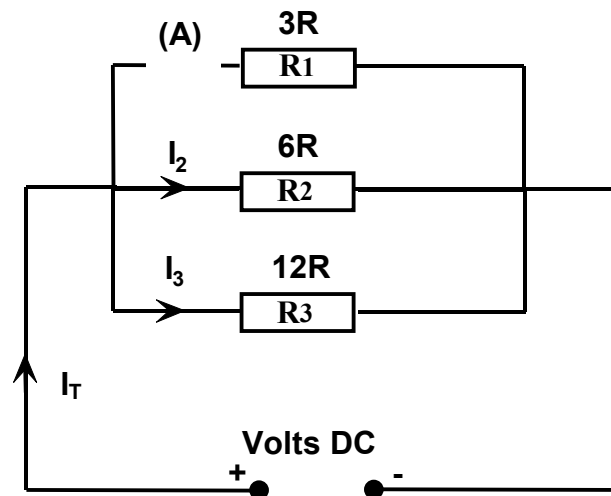


Figure 21.

Position **A** in Figure 21, has an open circuit fault which results in no current flow through resistor R_1 . This does **not** affect the other resistor currents I_2 and I_3 .

$$I_1 = \text{Zero Amps}$$

$$I_T = I_2 + I_3$$

Short Circuit Fault

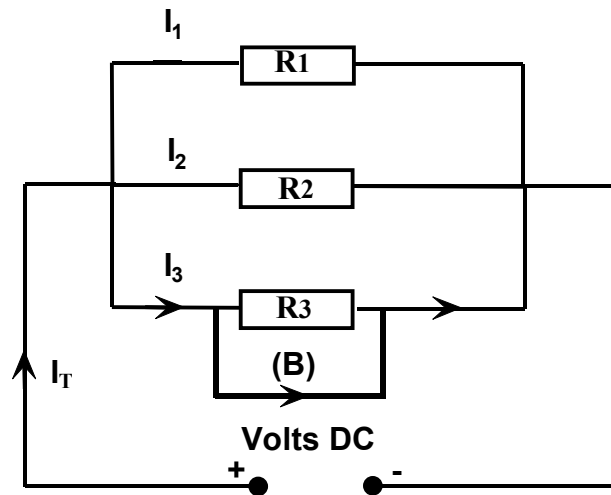


Figure 22.

Position **B** in Figure 22 has a short circuit fault across resistor R_3 . This results in excessive current flow through the short circuit.

$$I_1 = 0 \text{ Amps (under short circuit conditions)}$$

$$I_2 = 0 \text{ Amps (under short circuit conditions)}$$

$$I_3 = I_T \text{ Amps (under short circuit conditions)}$$

Warning:

A short-circuit in any resistor in a parallel circuit will result in excessive current flowing in that part of the circuit. This situation may result in the blowing of a fuse or the tripping of an MCB, otherwise the heat produced by that **excessive current** would cause damage.

Summary of Parallel Circuits

- There is only one voltage across all resistors in a parallel circuit.
- The total current I_T is equal to the sum of all the individual currents.

$$\blacksquare I_T = I_1 + I_2 + \dots + I_N$$

- The total resistance of a parallel circuit is **always less** than the resistance of the smallest resistor in the circuit.
- The total resistance is calculated by:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}$$

- (Applies to all resistors in parallel)

$$\bullet R_T = \frac{\text{Resistor value}}{\text{No. in parallel}} = \frac{R}{\text{No.}}$$

(Applies to equal values of resistors in parallel)

$$\bullet R_T = \frac{\text{Product}}{\text{Sum}} = \frac{R_1 \times R_2}{R_1 + R_2}$$

(Applies to two unequal, or two equal values of resistors in parallel)

- An open circuit in one resistor in a parallel circuit results in no current flowing in that resistor, but current flow in the other resistors is not affected.
- A short-circuit in any resistor in a parallel circuit will result in excessive current flowing in that part of the circuit. This situation may result in the blowing of a fuse or the tripping of an MCB, otherwise the heat produced by that excessive current would cause damage.

Example Questions

Q1. Calculate the total resistance of each of the following parallel-connected resistor banks:

- (a) $2\ \Omega$ and $6\ \Omega$.
- (b) $4\ \text{k}\Omega$, $8\ \text{k}\Omega$ and $8\ \text{k}\Omega$.
- (c) $6\ \text{M}\Omega$, $12\ \text{M}\Omega$ and $36\ \text{M}\Omega$.
- (d) $0\text{R}2$, $0\text{R}04$ and $0\text{R}006$.

Q2. Three resistors, having resistances of $4\text{R}8$, 8R and 12R , are connected in parallel and supplied from a 48V supply.

Calculate:

- (a) The current through each resistor.
- (b) The current taken from the supply.
- (c) The total resistance of the group.

Q3. Three resistors are connected in parallel across a supply of unknown voltage. Resistor 1 is $7\text{R}5$ and takes a current of $4\ \text{A}$. Resistor 2 is 10R and Resistor 3 is of unknown value but takes a current of $10\ \text{A}$.

Calculate:

- (a) The supply voltage.
- (b) The current through Resistor 2.
- (c) The value of Resistor 3.

Q4. Three parallel-connected busbars have resistances of $0\text{R}1$, $0\text{R}3$ and $0\text{R}6$, respectively, and in the event of a short circuit, they would be connected directly across a $400\ \text{V}$ supply.

Calculate:

- (a) The total resistance of the three busbars.
- (b) The total fault current.
- (c) The current through each busbar.

Kirchoff's Laws

Kirchoff's Current Law

Kirchoff's **Current Law** states that the sum of the currents entering a junction must be equal to the sum of the currents leaving the junction. Refer to Figure 23.

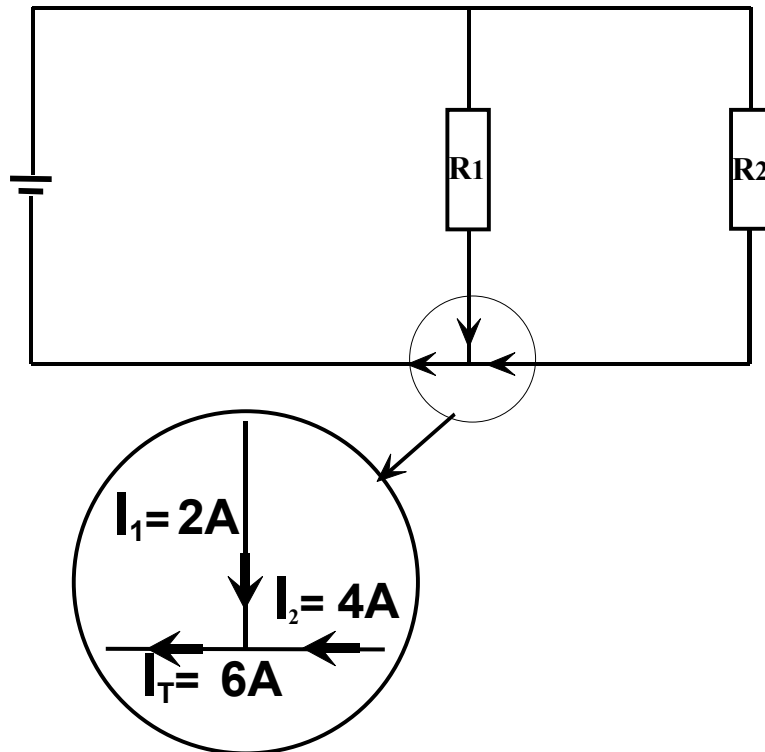


Figure 23.

The total current leaving the junction is 6amps.

The total current entering the junction is **4 Amps + 2 Amps = 6 Amps**.

Kirchoff's Voltage Law

Kirchoff's **Voltage Law** states that the sum of the **series volt drops** in a circuit equals the applied voltage. Refer to Figure 24.

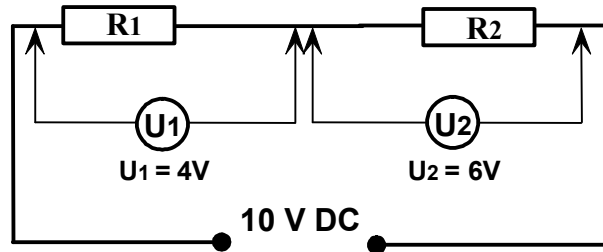


Figure 24.

$$\begin{aligned} \text{Applied Voltage} \quad U_A &= U_1 + U_2 \\ 10 \text{ Volts} &= 4 \text{ Volts} + 6 \text{ Volts} \end{aligned}$$

The sum of the series volt drops equals the applied voltage.

The Series - Parallel Circuit

In many electrical circuits, some components are connected in **series** so that the **same current** flows through them, while others are connected in **parallel** so that the **same voltage** is applied across them. Such a circuit is used where it is necessary to provide different values of current and / or voltage from one source of supply.

Series-parallel circuits are solved, by applying the basic rules of series circuits and parallel circuits, to obtain the total resistance.

Worked Example:

Refer to Figure 25. **First** find resistance of **parallel** part of circuit (R_p):

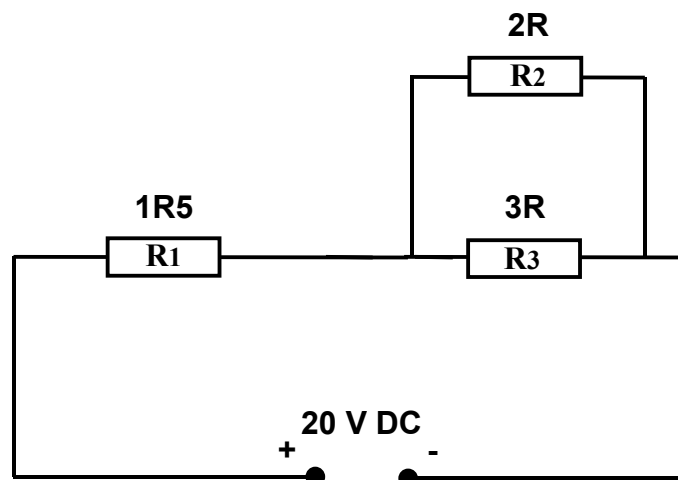


Figure 25.

$$R_p = \frac{\text{Product}}{\text{Sum}} = \frac{R_2 \times R_3}{R_2 + R_3} = \frac{2 \times 3}{2 + 3} = \frac{6}{5} = \underline{\underline{1R2}}$$

The circuit may then be redrawn as shown in Figure 26. R_p is the equivalent resistance of the combination of R_2 and R_3 in parallel.

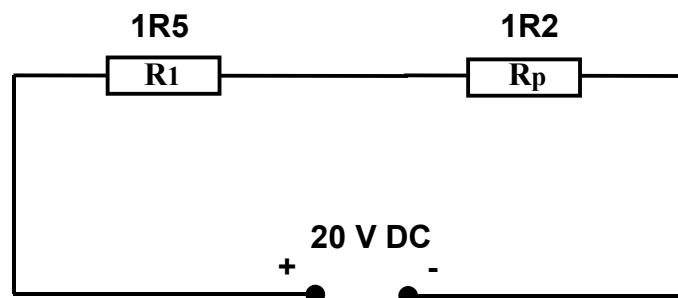


Figure 26.

From Figure 26 we can now continue with the calculation.

$$\begin{aligned} R_T &= R_1 + R_p \\ R_T &= 1.5 + 1.2 \\ R_T &= \underline{\underline{2.7 \Omega}} \end{aligned}$$

Now to find the volt drop across each resistor

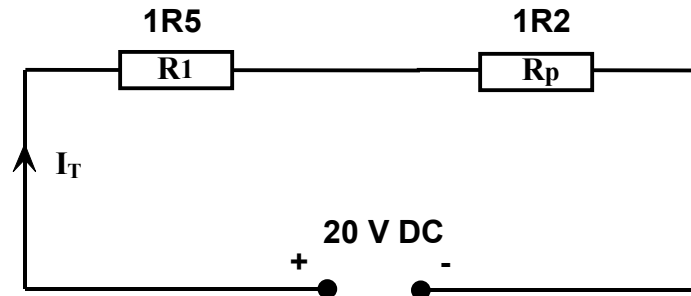


Figure 27.

Refer to Figure 27. From Ohm's Law we can now calculate the total circuit current.

$$\begin{aligned} I_T &= \frac{U_A}{R_T} \\ I_T &= \frac{20}{2.7} = \underline{\underline{7.41 \text{ Amps}}} \end{aligned}$$

Volt drop across R_1 ,

$$\begin{aligned} U_1 &= I_T \times R_1 \\ U_1 &= 7.41 \times 1.5 \\ U_1 &= \underline{\underline{11.11 \text{ Volts}}} \end{aligned}$$

Volt drop across R_p ,

$$\begin{aligned} U_p &= I_T \times R_p \\ U_p &= 7.41 \times 1.2 \\ U_p &= \underline{\underline{8.89 \text{ Volts}}} \end{aligned}$$

Check:

$$\begin{aligned} U_A &= U_1 + U_p \\ U_A &= 11.11 + 8.89 \\ U_A &= \underline{\underline{20 \text{ Volts}}} \end{aligned}$$

Now let us look at the original circuit again.

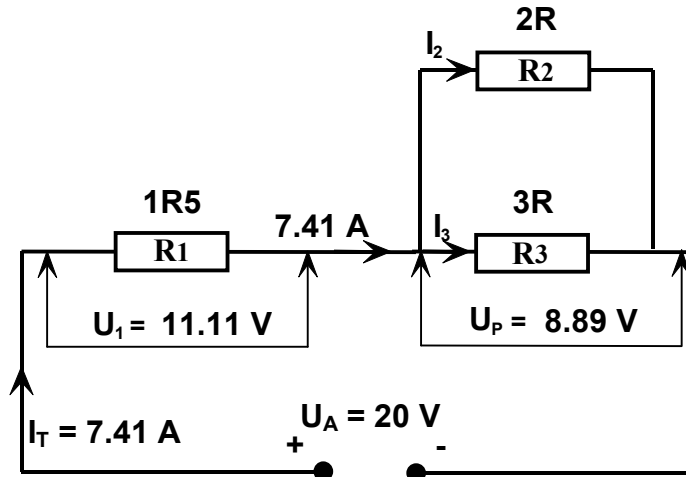


Figure 28.

Refer to Figure 28. The current through the 2R and 3R resistors will split up in inverse proportion to the value of each resistance. The **same volt drop** appears across the 2R and the 3R resistors as they are connected in **parallel**.

$$U_P = U_2 = U_3$$

To find the current through each resistor:

$$I_2 = \frac{U_2}{R_2}$$

$$I_2 = \frac{8.89}{2} = \underline{\underline{4.445 \text{ Amps}}}$$

$$I_3 = \frac{U_3}{R_3}$$

$$I_3 = \frac{8.89}{3} = \underline{\underline{2.96 \text{ Amps}}}$$

Check:

$$I_T = I_2 + I_3$$

$$I_T = 4.445 + 2.963$$

$$I_T = \underline{\underline{7.41 \text{ Amps}}}$$

Series - Parallel Circuit Layouts

What is the difference in the circuits below?

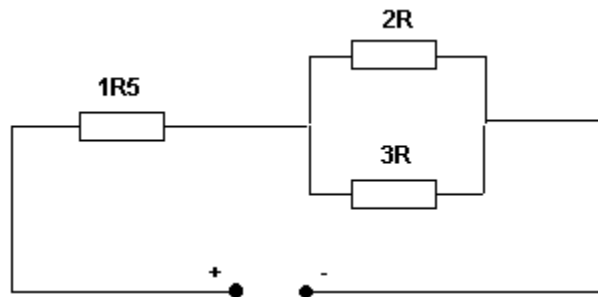


Figure 29

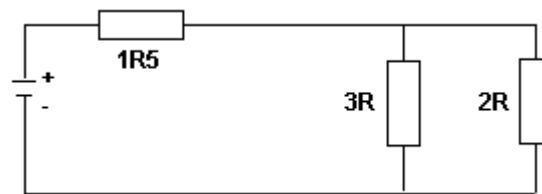


Figure 30

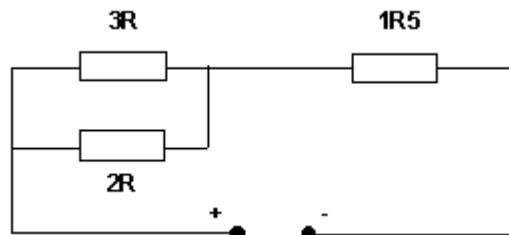


Figure 31

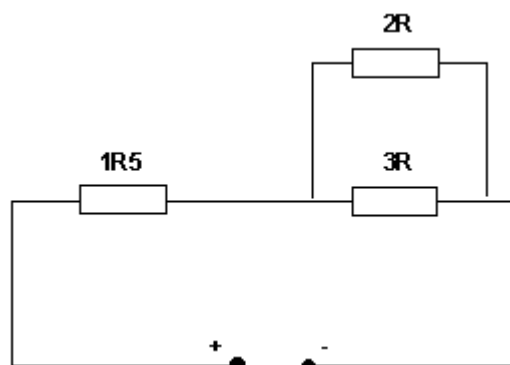
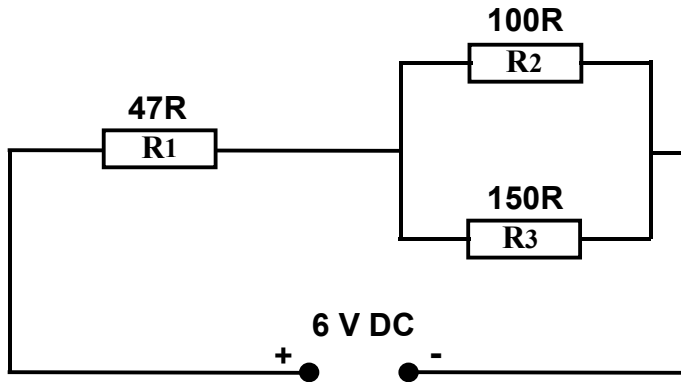


Figure 32

The resistor network arrangements are the same and the only difference is the layout of the drawings. The total current and the individual resistor currents are all the same. The volt drops across all parallel parts are also the same.

Example**Figure 33**

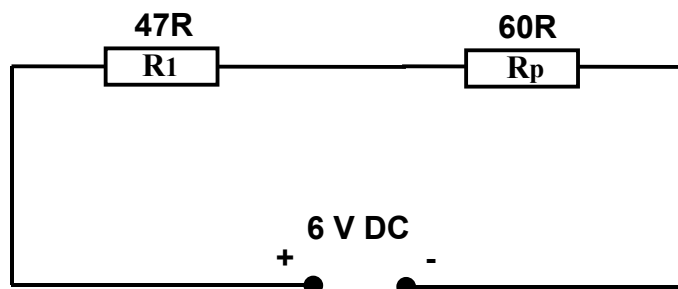
Refer to Figure 33.

- Find
- (1) Total resistance of circuit (R_T)
 - (2) Total Current (I_T)
 - (3) The currents
 - (a) I_2 through R_2
 - (b) I_3 through R_3

Solution

Start with Parallel resistors

$$\begin{aligned} \text{Equivalent } R_p &= \frac{\text{Product}}{\text{Sum}} = \frac{R_2 \times R_3}{R_2 + R_3} \\ R_p &= \frac{100 \times 150}{100 + 150} = \mathbf{60R} \end{aligned}$$

**Figure 34**

$$\begin{aligned} R_T &= R_1 + R_p \\ R_T &= 47 + 60 \\ R_T &= \mathbf{107R} \end{aligned}$$

$$I_T = \frac{U_A}{R_T}$$

$$I_T = \frac{6}{107}$$

$$I_T = \underline{\underline{\mathbf{0.056\ Amps\ or\ 56\ mA}}}$$

Volt drop across parallel resistor (U_P):

$$U_P = I_T \times R_p$$

$$U_P = 0.056 \times 60$$

$$U_P = \underline{\underline{\mathbf{3.36\ Volts}}}$$

Current I_2 , through R_2 :

$$I_2 = \frac{U_P}{R_2}$$

$$I_2 = \frac{3.36}{100}$$

$$I_2 = \underline{\underline{\mathbf{0.0336\ Amps\ or\ 33.6\ mA}}}$$

Current I_3 , through R_3 :

$$I_3 = \frac{U_P}{R_3}$$

$$I_3 = \frac{3.36}{150}$$

$$I_3 = \underline{\underline{\mathbf{0.0224\ Amp\ or\ 22.4\ mA}}}$$

Activity

Apprentices to set up the circuit shown in Figure 33 and:

1. Measure the resistance of the parallel resistors.
2. Measure the total resistance of the circuit.
3. Connect a 6 Volt supply.
4. Measure the voltage across the parallel resistors.
5. Measure the currents I_T , I_2 and I_3 .
6. Compare the measured values with calculations.

Activity

Refer to Figure 35. Apprentices to:

Connect three identical lamps as shown in Figure 35.

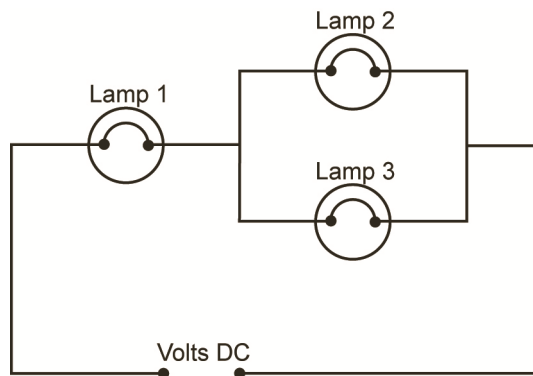
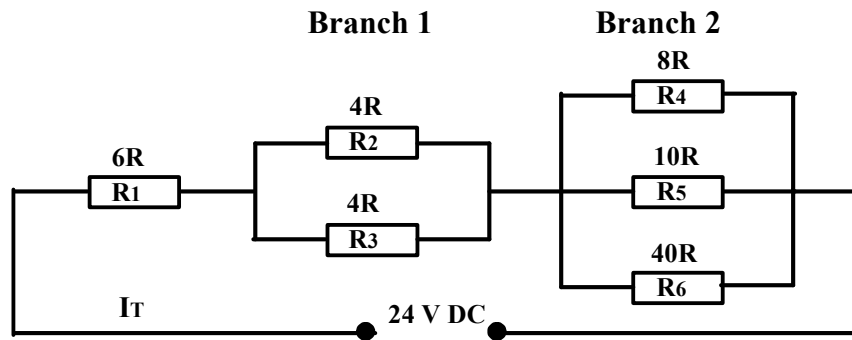


Figure 35.

1. What will happen with supply connected?
Observe Lamp 1, Lamp 2 and Lamp 3.
2. What will happen when Lamp 1 is removed?
3. With Lamp 1 back in position what will happen when Lamp 2 is removed?
4. With Lamp 2 back in position what will happen when Lamp 3 is removed?
5. Which Lamp(s) are in series with each other?
6. Which Lamp(s) are in parallel.

Example**Figure 36.**

- Refer to Figure 36. Find:
1. Total resistance of circuit (R_T)
 2. Total Current (I_T)
 3. Voltage drop across each resistor
 4. Current flow through each resistor.

Solution

Parallel Branch 1 Equivalent Resistance: R_{P1}

$$R_{P1} = \frac{R_1}{N_0} = \frac{4}{2} = \underline{\underline{2R}}$$

Parallel Branch 2 Equivalent Resistance: R_{P2}

$$\frac{1}{R_{P2}} = \frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_6}$$

$$\frac{1}{R_{P2}} = \frac{1}{8} + \frac{1}{10} + \frac{1}{40}$$

$$\frac{1}{R_{P2}} = \frac{5 + 4 + 1}{40}$$

$$\frac{1}{R_{P2}} = \frac{10}{40}$$

$$R_{P2} = \frac{40}{10} = \underline{\underline{4R}}$$

$$R_T = R_1 + R_{P1} + R_{P2}$$

$$R_T = 6 + 2 + 4$$

$$R_T = \underline{12R}$$

$$I_T = \frac{U_A}{R_T} = \frac{24}{12} = \underline{2 \text{ Amps}}$$

Figure 36 may now be redrawn as in Figure 37 and the circuit volt drops can be calculated.

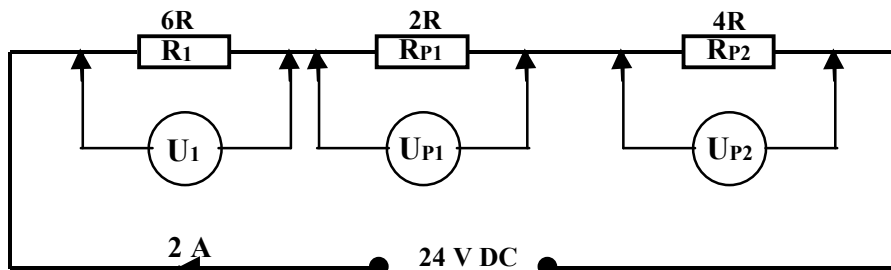


Figure 37.

Volt drop across R₁:

$$U_1 = I_T \times R_1$$

$$U_1 = 2 \times 6 = \underline{12 \text{ Volts}}$$

Volt drop across R_{P1}:

$$U_{P1} = I_T \times R_{P1}$$

$$U_{P1} = 2 \times 2 = \underline{4 \text{ Volts}}$$

Volt drop across R_{P2}:

$$U_{P2} = I_T \times R_{P2}$$

$$U_{P2} = 2 \times 4 = \underline{8 \text{ Volts}}$$

Original circuit

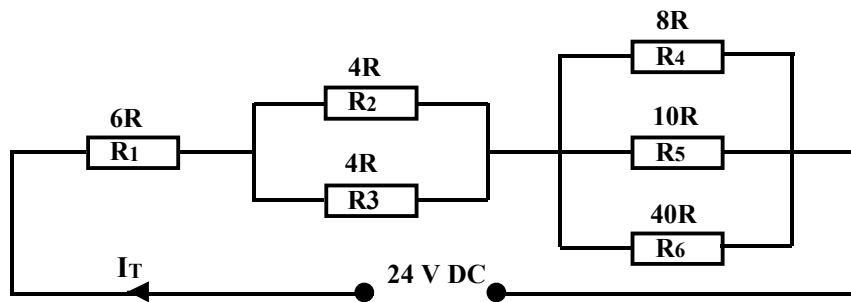


Figure 38.

To find the current through each resistor:

$$I_1 \text{ through } R_1 = \underline{\mathbf{2 \text{ Amps}}} \quad (\text{Also total current } I_T)$$

$$I_2 \text{ through } R_2 = \frac{U_{P1}}{R_2} = \frac{4}{4} = \underline{\mathbf{1 \text{ Amps}}}$$

I_3 through R_3 is equal to I_2 as they are equal resistors in parallel.

$$I_4 \text{ through } R_4 = \frac{U_{P2}}{R_4} = \frac{8}{8} = \underline{\mathbf{1 \text{ Amps}}}$$

$$I_5 \text{ through } R_5 = \frac{U_{P2}}{R_5} = \frac{8}{10} = \underline{\mathbf{0.8 \text{ Amps}}}$$

$$I_6 \text{ through } R_6 = \frac{U_{P2}}{R_6} = \frac{8}{40} = \underline{\mathbf{0.2 \text{ Amps}}}$$

Example Questions:

1. A $2R$ resistor is connected in series with a parallel bank consisting of a $6R$ and two $12R$ resistors. The supply voltage is 10 V .

Calculate:

- (a) The total resistance of the circuit.
- (b) The total current.
- (c) The current through each resistor.
- (d) The voltage drop across each resistor.

2. A $2.5R$ resistor is connected in series with a parallel bank, consisting of $7R$, $14R$, and $14R$ respectively. The current flow through the $2.5R$ resistor is 2 A .

Calculate:

- (a) The supply voltage.
- (b) The volt drop across the $7R$ resistor.

3. A $3R$ resistor is connected in series with two banks of parallel connected resistors. Bank A consists of a $20R$ and a $30R$ resistor, while bank B consists of three $15R$ resistors. The supply voltage is 60 V .

Calculate:

- (a) The total resistance of the circuit.
- (b) The total current.
- (c) The current through each resistor.
- (d) The voltage drop across each resistor.

4. Three banks of resistors are connected in series across a 230 V supply. Bank A consists of three resistors R_1 , R_2 and R_3 , each of resistance $60R$, connected in parallel. Bank B has two resistors, R_4 of resistance $40R$ and R_5 of resistance $120R$ connected in parallel. Bank C has three resistors, R_6 of resistance $50R$, R_7 of resistance $100R$ and R_8 of resistance $300R$ connected in parallel.

Calculate:

- (a) The total resistance of the circuit.
- (b) The total current
- (c) The current through each resistor.
- (d) The voltage drop across each resistor.

The Loading Effect of a Voltmeter

Measuring Voltage in a Low Resistance Circuit

Refer to Figure 39.

This series circuit was covered earlier. Note the voltdrops across R_1 and R_2

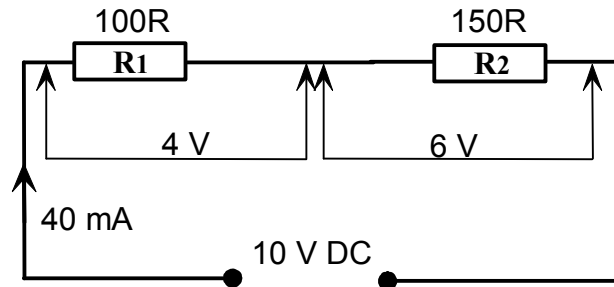


Figure 39.

Refer to Figure 39.

An **analogue** voltmeter is shown connected across R_1 . This changes the resistance of the circuit.

Let us examine the “**loading effect**” of using this voltmeter to measure the voltage across the 100R resistor.

Assume the meter has a resistance of 20,000R.

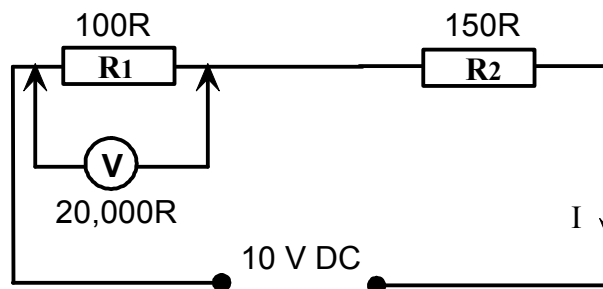


Figure 40.

Total resistance of parallel branch (100R resistor and the meter resistance of 20,000R):

$$R_p = \frac{R_1 \times R_M}{R_1 + R_M}$$

$$R_p = \frac{20,000 \times 100}{20,100} = \frac{2,000,000}{20,100}$$

$$R_p = \underline{\underline{99.5 \Omega}}$$

Total Resistance of Circuit

$$R_T = R_P + R_2$$

$$R_T = 99.5 + 150$$

$$R_T = \underline{\underline{249.5}}$$

$$I = \frac{U_A}{R_T}$$

$$I = \frac{10}{249.5}$$

$$I = \underline{\underline{40.1 \text{ mA}}} = \underline{\underline{0.0401 \text{ Amps}}}$$

The new volt drop across the 100R is:

$$U_1 = I_T \times R_P$$

$$U_1 = 0.0401 \times 99.5$$

$$U_1 = \underline{\underline{3.98 \text{ Volts}}}$$

When an analogue meter is used to measure voltage across a low value resistor, its “loading effect” on the circuit is minimal and can be ignored in most situations.

Measuring Voltage in a High Resistance Circuit

Refer to Figure 41.

Find the current through and the volt drop across (a) the 30k resistor and (b) the 20k resistor.

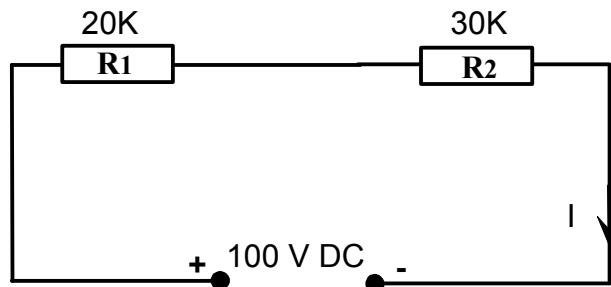


Figure 41.

Solution: $R_T = R_1 + R_2$

$$R_T = 20,000 + 30,000$$

$$R_T = \underline{\underline{50,000R}}$$

$$I = \frac{U_A}{R_T} = \frac{100}{50,000} = \underline{\underline{0.002 \text{ Amps}}}$$

Volt Drop across 20k resistor

$$U_1 = I \times R_1$$

$$U_1 = 0.002 \times 20,000$$

$$U_1 = \underline{\underline{40 \text{ Volts}}}$$

Volt Drop across 30k resistor

$$U_2 = I \times R_2$$

$$U_2 = 0.002 \times 30,000$$

$$U_2 = \underline{\underline{60 \text{ Volts}}}$$

Refer to Figure 42.

An **analogue** voltmeter is shown connected across R_1 . This changes the resistance of the circuit.

Let us examine the “**loading effect**” of using this voltmeter to measure the voltage across the 20,000R resistor.

Assume the meter has a resistance of 20,000R.

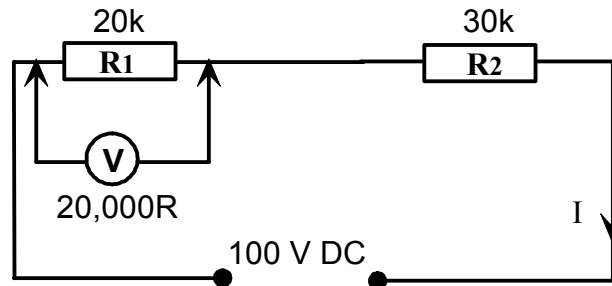


Figure 42.

$$\frac{1}{R_P} = \frac{1}{R_M} + \frac{1}{R_2}$$

$$\frac{1}{R_P} = \frac{1}{20,000} + \frac{1}{20,000} = \frac{2}{20,000}$$

$$\frac{1}{R_P} = \frac{1}{10,000}$$

$$R_P = \underline{\underline{10,000R}}$$

$$R_T = R_1 + R_P$$

$$R_T = 30,000 + 10,000$$

$$R_T = \underline{\underline{40,000R}}$$

$$I_T = \frac{U_A}{R_T} = \frac{100}{40,000}$$

$$I_T = \underline{\underline{0.0025 \text{ Amps}}}$$

Volt Drop across parallel branch:

$$U_1 = I_T \times R_p$$

$$U_1 = 0.0025 \times 10,000$$

$$U_1 = \underline{\underline{25 \text{ Volts}}}$$

Volt Drop across 30k resistor:

$$U_2 = I_T \times R_2$$

$$U_2 = 0.0025 \times 30,000$$

$$U_2 = \underline{\underline{75 \text{ Volts}}}$$

Summary of measurements when using an analogue meter in a high resistance circuit.

	Total Current	Voltage across R ₁	Voltage across R ₂
Without Meter	0.002 Amps	40 Volts	60 Volts
With Meter	0.0025 Amps	25 Volts	75 Volts
% Change	25% Increase	37.5% Decrease	25% Increase

Conclusions

- A wrong voltage is recorded.
- There is a “loading effect” on the circuit.

The loading effect is minimised, by using a voltmeter having a resistance much greater than the resistor across which the voltage is being measured. It will be noticed that in such a case, there is a very slight **effect** on the circuit, but in the high resistance circuit, connecting the voltmeter had a much greater **effect** on the circuit.

Cells and Batteries

Cells

A cell generates electrical energy from an internal chemical reaction. It consists of two conducting materials called electrodes, which are immersed in a liquid or paste called an electrolyte. The internal chemical reaction in a cell results in a separation of electric charges in the form of ions and free electrons. As a result, the two electrodes have a difference of potential between them, which provides an electro-motive-force (EMF).

A standard cell has an open circuit voltage of **1.5 Volts**. Other special cells are manufactured having voltages of 1.2 Volts, 2 Volts and 3 Volts. It is most important that a cell of the correct voltage is selected.

Primary Cells

Cells, which must be replaced when the chemical reaction is exhausted, are collectively called *Primary Cells* (Dry Cell).

Secondary Cells

A cell, which can be repeatedly recharged is called a *Secondary Cell*, the most familiar of which is the lead-acid type used in motor vehicles.

Batteries

A battery is simply a group of two or more similar cells connected together.

A typical car battery is shown below.

Typical 12 V battery
consisting of
6 lead / sulphuric acid **cells**
connected in series

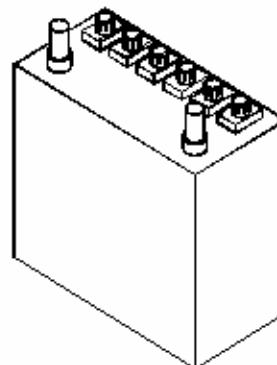


Figure 43.

Cells in Series

When **similar cells** are connected in series, the total output voltage can be calculated by adding the individual cell voltages. Three 1.5 Volt cells connected in series will give an open circuit voltage of:

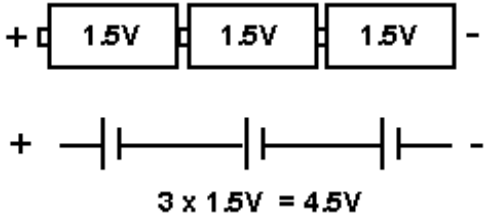
$$1.5 \text{ V} + 1.5 \text{ V} + 1.5 \text{ V} = 4.5 \text{ V}$$


Figure 44.

$$\text{Terminal Voltage } U_T = U_1 + U_2 + U_3$$

The maximum **current** that can be supplied to a load will be the maximum current of any one of the cells as they are connected in **series**. If a battery has a “weak” cell, that cell will determine the maximum current flow.

$$I_T = I_1 = I_2 = I_3$$

Cells are connected in **series** to achieve a higher **voltage**. It is important that **correct polarity** is observed when installing cells or batteries.

Cells in Parallel

When **similar cells** are connected in parallel, the total output voltage will be the open circuit voltage of any individual cell.

When three 1.5 Volt cells are connected in parallel as shown in Figure 45, the output voltage will be the open circuit voltage of any one of the three cells.

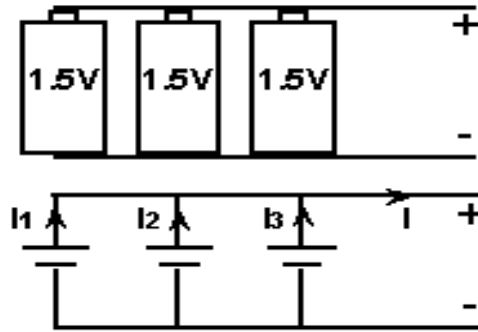


Figure 45.

$$\begin{array}{lcl} \text{Voltage of one cell} & = & 1.5 \text{ Volts} \\ \text{Voltage of three cells in parallel} & = & 1.5 \text{ Volts} \end{array}$$

$$U_T = U_1 = U_2 = U_3$$

The maximum **voltage** that can be applied to a load will be the maximum voltage of any one of the cells as they are connected in **parallel**. If a battery has a “weak” cell, that cell voltage will be lower than the voltage of the other cells.

The maximum output current will be the sum of the individual cell currents.

$$I_T = I_1 + I_2 + I_3$$

Cells are connected in **parallel** to achieve a higher output **current**.

Electromotive Force (EMF or E)

The EMF of a cell or battery is measured in volts. It is the maximum force available from a cell or battery to produce current flow in a circuit.

Internal Resistance

The amount of current a cell can deliver depends on a number of factors. Basically the bigger a cell is the more current it can deliver. There is always an **internal opposition** to the amount of current a cell can deliver and this is called “internal resistance”. (**Symbol r**)

The internal resistance is inside the cell, but is usually represented as in Figure 46.

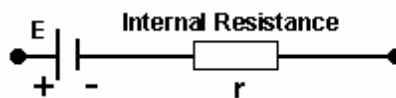


Figure 46.

Terminal Voltage of a Cell

The Terminal Voltage (U) of a cell is measured in volts.

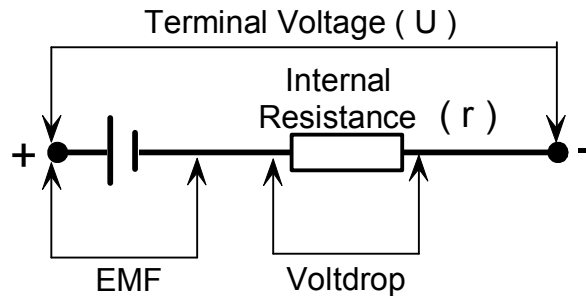


Figure 47

Terminal Voltage	U	=	EMF	-	Volt Drop
Terminal Voltage	U	=	E	-	I x r

The greater the current a cell delivers, the greater the volt drop (I x R) and the Terminal Voltage (U) of the cell will be reduced accordingly.

The effect of Internal Resistance

The EMF of a cell is measured at 1.5 Volts. When connected to a load of 400 mA its terminal voltage is measured at 1.3 Volts. The internal resistance (r) can now be calculated.

$$\begin{aligned} \text{Volt Drop across } r &= \text{EMF} - \text{Terminal Voltage} \\ &= 1.5 - 1.3 \\ &= 0.2 \text{ Volt} \end{aligned}$$

The Internal Resistance is equal to the Volt Drop across the Internal Resistance divided by the Load Current.

$$r = \frac{\text{Volt Drop}}{\text{Load Current}} = \frac{0.2}{0.4} = \underline{\underline{0.5R \text{ or } 0R5}}$$

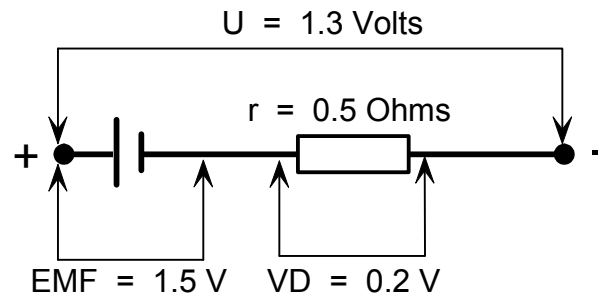


Figure 48.

Internal Resistance of Cells connected in Series:

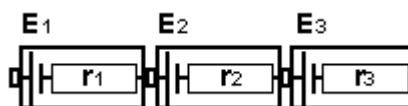


Figure 49.

Terminal voltage of 3 cells each 1.5 V connected in series is:

$$1.5 \text{ V} + 1.5 \text{ V} + 1.5 \text{ V} = 4.5 \text{ Volts}$$

When cells are connected in series the internal resistances are also in series. The total internal resistance of the battery is calculated as follows:

$$r_T = r_1 + r_2 + r_3$$

Internal Resistance of Cells connected in Parallel:

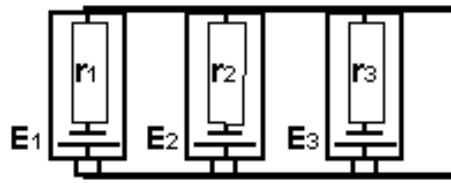


Figure 50.

When cells are connected in parallel the internal resistances of the cells are also in parallel. The total internal resistance of the battery is calculated as follows:

$$\frac{1}{r_T} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

In a parallel circuit the total resistance is always less than the smallest resistor in the circuit.

When cells are connected in parallel the internal resistance is reduced, which allows the cells supply more current.

Battery and Cell Usage

Batteries and cells are used in the automobile industry, fire / security / intruder alarm panel systems, emergency / exit lighting systems, multi-meters, calculators, cameras, clocks, watches, computers, lamps, torches etc.

Battery and Cell Warning

1. Manufacturer's instructions should be strictly adhered to.
2. Exercise care with disposal of batteries (do not dispose in fire).
3. Observe polarity when installing.
4. Do not short-circuit a battery (some can deliver hundreds of amps when shorted and will explode).
5. The handling of some batteries requires protective clothing suitable for the purpose.

Activity

Connect cells in series and in parallel and measure the output voltages.

Resistance of Cable Conductors

One of the more important tasks the electrician has to perform is the selection of a suitable cable for a particular job. There are several points to be considered before the final selection is made. In many cases, one may find several different types of cable suitable for a given application. Most cables encountered in electrical installation work comprise an outer sheath for mechanical protection, inner insulation for electrical isolation, and the conductor itself, which carries the circuit current.

See Figure 51.

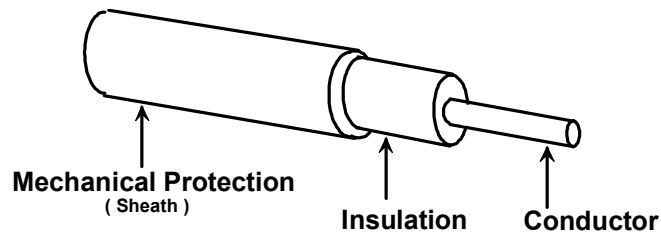


Figure 51.

Conductors

The **conductive part** of a cable is normally **copper**, but aluminium may be used for reasons of lightness or economy. The **current carrying capacity** of a cable is largely determined by the **cross-sectional area** (CSA) of its conductors. Cables are available in sizes ranging from 1mm² to 630 mm² (copper) or 16 mm² to 1,000 mm² (aluminium).

$$\text{Cross sectional area } a = \frac{\pi d^2}{4} \quad \text{or} \quad \pi r^2$$

Cables are sized by the cross sectional area of their conductors. For example a 2.5 mm² cable is generally used for wiring sockets in houses. This 2.5 mm² refers to the **cross sectional area** of the cable **conductors**. The diameter of the conductor can be calculated as follows:

$$\text{CSA} = \frac{\pi d^2}{4} \quad \left\{ \begin{array}{l} a = \text{Cross-Sectional Area} \\ \pi = 3.14 \\ d = \text{diameter of conductor} \end{array} \right.$$

$$\text{CSA} \times 4 = \pi d^2$$

$$d^2 = \frac{\text{CSA} \times 4}{\pi} = \frac{2.5 \times 4}{3.14} = 3.18$$

$$d = \sqrt{3.18}$$

$$d = \underline{\underline{1.78 \text{ mm}}} \text{ (A } 2.5\text{mm}^2 \text{ CSA conductor has a diameter of } 1.78 \text{ mm).}$$

Factors affecting the Resistance of a Conductor

The following three factors determine the resistance of a conductor at a constant temperature:

1. Resistivity of conductor material.
2. Length of conductor.
3. Cross sectional area of conductor.

Resistivity

The resistance between opposite faces of a cube of material is called **resistivity** or **specific resistance**. It is measured in terms of the resistance of a cubic metre of the material. This resistance will depend on the type of material involved. A material which has a low resistance is a good conductor (high conductivity). A material, which has a high resistance, is a poor conductor (low conductivity). Copper has a lower resistance than aluminium for example.

The resistivity of a material is defined as the resistance measured between the opposite faces of a unit cube of the material.

Symbol = Greek letter ρ (pronounced Rho)

When we want to compare current carrying properties of conductor materials, we must compare equal quantities. The resistance between 2 opposite faces of the cubes will give us the resistivity of the material.

See Figure 52.

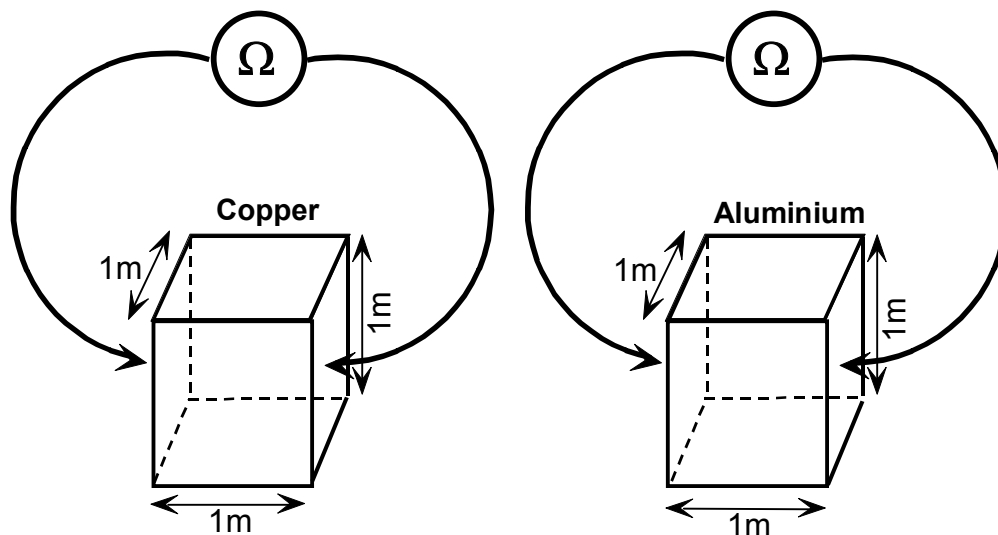


Figure 52.

The values measured will be very low:

Copper: 0.0172 $\mu\Omega\text{m}$. (micro-ohms per cubic metre)
 Aluminium: 0.0285 $\mu\Omega\text{m}$

The cube of copper measures approximately $\frac{2}{100}$ — of 1 Micro ohm

The cube of aluminium measures approximately $\frac{3}{100}$ of 1 Micro ohm

Since copper has the **lower resistivity** we say it is **a better conductor** than aluminium.
 The composition of a material determines its resistivity value.

Resistance (R) is proportional to Resistivity (ρ)

$$R \propto \rho$$

Resistivity of Common Conductors Materials

The table below shows clearly why copper is so widely used as a conductor material. It is second only to silver in its resistivity value. Aluminium is more resistive than copper, but it is lighter and cheaper. This has led to its increasing use as a conductor material.

<u>Material</u>	<u>Resistivity:</u>		
	<u>$\mu\Omega\text{m}$</u>	<u>$\mu\Omega\text{cm}$</u>	<u>$\mu\Omega\text{mm}$</u>
Silver	0.0163	1.63	16.3
Copper (annealed)	0.0172	1.72	17.2
Copper (hard drawn)	0.0178	1.78	17.8
Aluminium	0.0285	2.85	28.5
Brass	0.06-0.09	6-9	60-90
Iron	0.100	10.0	100
Tin	0.114	11.4	114
Lead	0.219	21.9	219
Mercury	0.958	95.8	958

Length of Conductor

The resistance of a conductor depends on its length. If we increase the length of conductor leaving the face area the same, its resistance will increase.

See Figure 53.

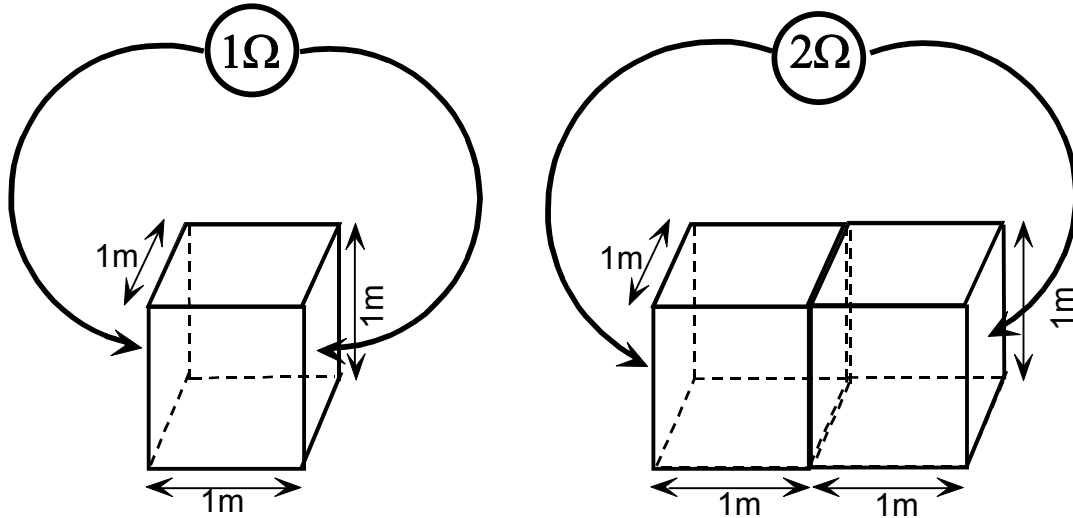


Figure 54.

If the length of conductor is doubled, its resistance is doubled. Consider this to be the same as two equal resistors in series. The resistance of a conductor is directly proportional to its length. If the length of a conductor is halved its resistance is halved.

$$R \propto l$$

Cross-Sectional Area of a Conductor

The resistance of a conductor depends on its cross-sectional area. If we increase the CSA of conductor leaving the length the same, its resistance will decrease.

See Figure 55.

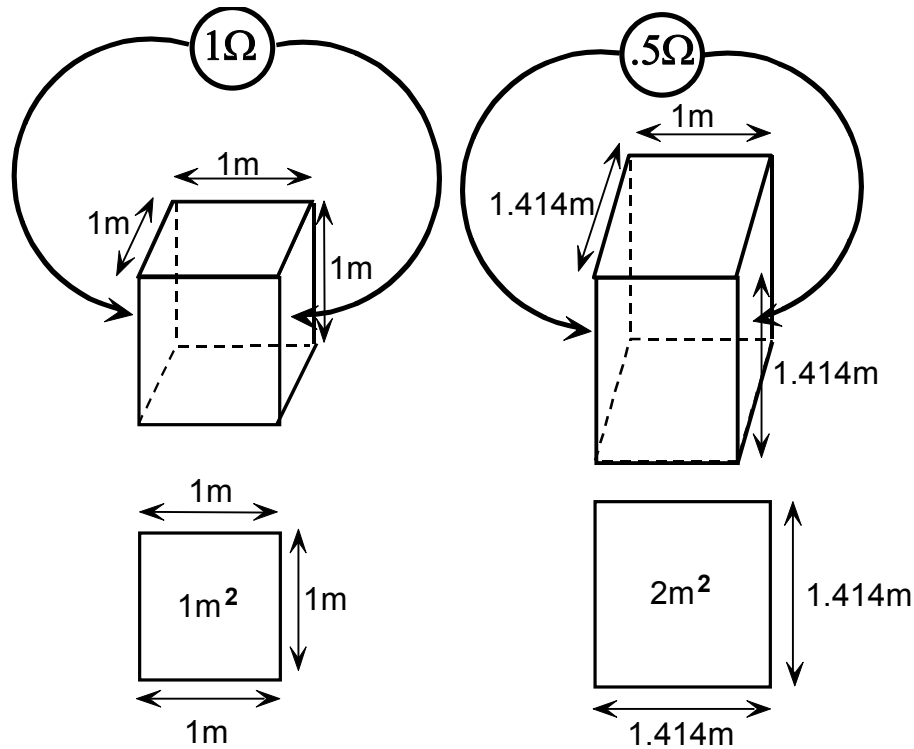


Figure 55.

If the CSA of the conductor is doubled, its resistance is halved. Consider this to be the same as two equal resistors in parallel. The resistance of a conductor is inversely proportional to its cross-sectional area.

$$R \propto \frac{1}{a}$$

The resistance of a conductor is directly proportional to its length and resistivity, and inversely proportional to its cross sectional area.

$$R = \frac{\rho l}{a}$$

Where: R = Conductor resistance
 ρ = Resistivity
 l = Length of conductor
 a = Cross-sectional area of conductor

Note: R and ρ must be in the same units
 ρ, l and a must be in the same units

Transposition of Formulae:

$$R = \frac{\rho l}{a} \quad (\text{to find Resistance})$$

cross-multiplying $Ra = \rho l$

$$a = \frac{\rho l}{R} \quad (\text{to find area})$$

$$\rho = \frac{Ra}{l} \quad (\text{to find resistivity})$$

$$l = \frac{Ra}{\rho} \quad (\text{to find length})$$

Example 1:

Calculate the resistance of 500 metres of 10 mm² annealed copper conductor if ρ for the material is 0.0172 $\mu\Omega\text{m}$.

Solution

$$R = \frac{\rho l}{a} \quad \rho, l \text{ and } a \text{ must be expressed in the same units.}$$

$$l = 500 \text{ metres} = 500 \times 10^3 \text{ mm}$$

$$a = 10 \text{ mm}^2$$

$$\rho = 0.0172 \mu\Omega\text{m} = 0.0172 \times 10^{-6} \Omega\text{m} = 0.0172 \times 10^{-6} \times 10^3 \Omega\text{mm}$$

$$R = \frac{\rho l}{a}$$

$$R = \frac{0.0172 \times 10^{-6} \times 10^3 \times 500 \times 10^3}{10}$$

$$R = \frac{0.0172 \times 500}{10}$$

$$R = \frac{1.72 \times 5}{10}$$

$$R = \frac{8.60}{10}$$

$$R = \underline{\underline{0.86 \Omega}}$$

Example 2:

A roll of 1.5 mm² PVC cable has a resistance of 0.6 Ω. If the resistivity of the material is 0.0172 μΩm, calculate the length of the cable.

Solution

$$R = \frac{\rho l}{a}$$

$$l = \frac{Ra}{\rho}$$

$$R = 0.6 \Omega.$$

$$\rho = 0.0172 \mu\Omega\text{m} = 0.0172 \times 10^{-6} \Omega\text{m} = 0.0172 \times 10^{-6} \times 10^3 \Omega\text{mm}$$

$$a = 1.5 \text{ mm}^2$$

$$l = \frac{Ra}{\rho}$$

$$l = \frac{0.6 \times 1.5}{0.0172 \times 10^{-6} \times 10^3}$$

$$l = \frac{0.9 \times 10^3}{0.0172}$$

$$l = 52.33 \times 10^3 \text{ mm}$$

$$l = \underline{\underline{52.33 \text{ metres}}}$$

Example 3:

A roll of cable is 40 metres long, its conductor diameter is 1.78 mm and its resistivity is 0.0172 $\mu\Omega\text{m}$. Calculate the resistance of the cable.

Solution

$$a = \frac{\pi d^2}{4}$$

$$a = \frac{3.14 \times 1.78 \times 1.78}{4}$$

$$a = 2.488 \text{ mm}^2$$

$$a = 2.5 \text{ mm}^2 \text{ (rounded to one decimal place)}$$

$$R = \frac{\rho l}{A}$$

$$\rho = 0.0172 \mu\Omega\text{m} = 0.0172 \times 10^{-6} \Omega\text{m}$$

$$l = 40 \text{ metres}$$

$$a = 2.5 \text{ mm}^2 = 2.5 \times 10^{-6} \text{ m}^2$$

$$R = \frac{0.0172 \times 10^{-6} \times 40}{2.5 \times 10^{-6}}$$

$$R = \frac{0.688}{2.5}$$

$$R = \underline{\underline{0.275 \Omega}}$$

Temperature Coefficient of Resistance

The temperature coefficient of resistance of a material is, the change in resistance of the material, resulting from a change in temperature of one degree centigrade.

- The resistance of most **conductor** materials vary with temperature change.
- The resistance of most conductor materials (including copper), increase as their temperature increases and are said to have a **positive temperature coefficient** (P.T.C.) of resistance.
- The resistance of carbon decreases as its temperature increases and it is said to have a **negative temperature coefficient** (N.T.C.) of resistance.
- Most insulator materials have a negative temperature coefficient of resistance.
- **Symbol for temperature coefficient of resistance is α (pronounced alpha).**